

A Renormalizable Supersymmetric SU(5) Model

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ABSTRACT: In the Supersymmetric SU(5) Model of Unification with the Missing Partner Mechanism, we present a renormalizable model using the Georgi-Jarlsog mechanism to describe the fermion masses and mixing. At the meantime the proton decay rates are also suppressed to satisfy the experimental data.

KEYWORDS: Unification, Supersymmetry, Fermion masses, Proton decay

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1 Introduction

A Grand Unified Theory (GUT) model need to realize the unification of the gauge couplings and give the correct fermion masses and mixing. In the SU(5) Supersymmetric (SUSY) GUT (SGUT) models, the unification of the gauge couplings [1, 2] can be achieved by taking threshold effects into account. The correct fermion masses and mixing, however, cannot be given in the minimal version of the SU(5) model which contains only $5 + \bar{5} + 24$ in the Higgs sector. In addition, the threshold effects in realizing the gauge coupling unification constrain the spectrum of the entire model[3]. In the minimal version of the SU(5) model, these constraints are quite strong. The resulting color-triplet Higgsino masses of the $5 + \bar{5}$ are rather low, which induce too rapid proton decay rates to be acceptable experimentally[4]. A related issue is the doublet-triplet splitting problem which requires a pair of nearly massless weak doublets to break the electroweak symmetry.

There are many efforts to solve the above problems. To give the correct fermion masses and mixing in the non-SUSY SU(5) model, the Georgi-Jarlskog mechanism (GJM)[5] can be used by introducing extra Higgs of $\overline{45}$. In the SUSY version of the GJM, Higgs superfields of $45 + \overline{45}$ need to be added[6]. Proton decay can be suppressed by the cancelation of different color-triplet pairs of Higgsino. However, the up-type quarks get masses not only from the original Yukawa couplings of $5 - 10_F - 10_F$ (the subscript F stands for fermion or matter) but also from the newly introduced Yukawa couplings of $45 - 10_F - 10_F$. This may induce more unobservable parameters in the up-type Yukawa couplings[7], although proton decay rates can be further suppressed since the relation is weakened between the fermion masses and the dimension-5 operators generated by the color-triplet Higgsinos.

People usually use the Missing Partner Mechanism (MPM)[8, 9] to resolve the doublet-triplet splitting problem. In the MPM a pair of $50 + \overline{50}$ Higgs are introduced which contain a pair of color-triplet Higgs without new weak-doublet Higgs. Instead of 24, a 75-Higgs is used to break the SU(5) symmetry. By introducing a U(1) symmetry[10], the suitable superpotential can be written to generate a pair of massless weak doublets. All Higgs superfields but a pair of weak doublets are heavy so that at low energy the model recover to be the Minimal SUSY Standard Model (MSSM).

In the present work we will give a realistic model by applying the GJM and the MPM simultaneously. The Higgs sector contains $1, 5 + \bar{5}, 45 + \overline{45}, 50 + \overline{50}, 75$ multiplets of SU(5), while the matter sector remains the same as in the minimal model. All the couplings are renormalizable in the present model, unlike in [10] where high-order couplings are used to generate fermion masses through the Frogatt-Nielsen mechanism[11]. By introducing two U(1) symmetries and giving the 45 different U(1) charges from the 5's, the up-type quarks get masses from the 5 only. This eliminates the extra couplings in [6] and makes the model rather predictable.

We will present the model explicitly. Then we carry the analysis on the GUT spectrum, calculate the constraints imposed by gauge coupling unification, study the fermion masses and proton decay, and conclude.

Table 1. $U_S(1)$ and $U_P(1)$ quantum numbers for the Higgs superfields.

Higgs	5	$\bar{5}$	45	$\bar{45}$	5'	$\bar{5}'$	50	$\bar{50}$	50'	$\bar{50}'$	75	S	P
$U_S(1)$	h	$-q-h$	$q+h$	$-q-h$	$q+h$	$-h$	$q+h$	$-h$	h	$-q-h$	0	$-q$	0
$U_P(1)$	τ	$-\sigma$	σ	$-\sigma$	τ	$-\sigma$	σ	$-\tau$	σ	$-\tau$	0	$-\tau+\sigma$	$\tau-\sigma$

2 The Model and the Spectrum

In the GUT models with MPM to realize the doublet-triplet splitting, extra Higgs of $50 + \bar{50}$ are added. To protect the gauge coupling evolution perturbatively below the GUT scale, these Higgs need to be as heavy as around the Planck scale. This is realized by introducing a $U_P(1)$ symmetry breaking just below the Planck scale. Furthermore, for the model to recover the MSSM as all but one pair of Higgs doublets are massless at the GUT scale, another symmetry $U_S(1)$ is added to break at a scale below the GUT scale. The Higgs sector contain two singlets, two pairs of $5 + \bar{5}$, two pairs of $50 + \bar{50}$, and one 75 to break the SU(5). This is in accord with the improved MPM of [13]. In addition, a pair of $45 + \bar{45}$ is needed to generate fermion masses and mixing through the GJM. We assign the $U_S(1)$ and $U_P(1)$ charges for the Higgs multiplets as in Table I.

The superpotential of Higgs sector is

$$\begin{aligned}
W = & -\frac{\sqrt{2}}{6}\lambda A 75_{kl}^{ij} 75_{ij}^{kl} + \lambda 75_{kl}^{ij} 75_{in}^{km} 75_{mj}^{nl} \\
& + \frac{m}{2} 45_k^{ij} \bar{45}_{ij}^k + \frac{3}{2\sqrt{2}} e \bar{5}_i 75_{jk}^{im} 45_m^{jk} + \frac{\Delta}{\langle S \rangle} S 5^i \bar{5}'_i \\
& + \frac{\sqrt{3}}{4\sqrt{2}} a 5^i 75_{lm}^{jk} \bar{50}_{ijk}^{lm} + \frac{\sqrt{3}}{4\sqrt{2}} b \bar{5}_i 75_{jk}^{lm} 50_{lm}^{ijk} + \frac{\sqrt{3}}{4\sqrt{2}} b' 5^i 75_{lm}^{jk} \bar{50}'_{ijk}^{lm} + \frac{\sqrt{3}}{4\sqrt{2}} a' \bar{5}'_i 75_{jk}^{lm} 50_{lm}^{ijk} \\
& + \frac{3\sqrt{3}}{2\sqrt{2}} f \bar{45}_{ij}^n 75_{kl}^{im} 50_{nm}^{jkl} + \frac{M_1}{12\langle P \rangle} P 50_{lm}^{ijk} \bar{50}'_{ijk}^{lm} + \frac{M_2}{12\langle P \rangle} P 50_{lm}^{ijk} \bar{50}_{ijk}^{lm},
\end{aligned} \tag{2.1}$$

where the coefficients are chosen for later convenience. We demand all the trilinear couplings are of order one to avoid fine-tuning. The mechanism of breaking the U(1)'s can be found in e.g. [12]. Also, we will not discuss whether these U(1) are global [10] or anomalous[13] which are both used in the literature.

Just below the Planck scale, the $U_P(1)$ symmetry is broken when the SU(5) singlet $P(0, -\sigma + \tau)$ obtain a vacuum expecting value (VEV) $\langle P \rangle$. This leads the $50, \bar{50}, 50', \bar{50}'$ to be heavy. The SU(5) symmetry is broken when the Standard Model (SM) singlet of the 75 obtains a VEV A , while the SU(5) singlet $S(-q, \sigma - \tau)$ obtain a vacuum value $\langle S \rangle$ to break the $U_S(1)$ symmetry at a lower scale.

At the GUT scale the heavy Higgs $50, \bar{50}, 50'$ and $\bar{50}'$ have been integrated out, we can get the spectrum of the Higgs multiplets in Table 2. The spectrum is to be constrained by the requirement of gauge coupling unification through the threshold effects. We have neglected some small effects of the 50's on the $45 + \bar{45}$.

The mass matrix for the color-triplet Higgs multiplets is

$$M_T = \begin{array}{c|ccc} & T_5 & T_{45} & T_{5'} \\ \hline \bar{T}_5 & 0 & -eA & -\frac{bb'A^2}{M_1} \\ \bar{T}_{45} & 0 & m & -\frac{fb'A^2}{M_1} \\ \bar{T}_{5'} & -\frac{aa'A^2}{M_2} & 0 & \Delta, \end{array} \tag{2.2}$$

and that for the weak-doublets is

$$M_2 = \begin{array}{c|ccc} & H_5 & H_{45} & H_{5'} \\ \hline \bar{H}_5 & 0 & \sqrt{3}eA & 0 \\ \bar{H}_{45} & 0 & m & 0 \\ \bar{H}_{5'} & 0 & 0 & \Delta. \end{array} \tag{2.3}$$

Diagonalizing M_2 gives two pair of heavy weak-doublets with masses

$$\begin{aligned}
M_+ &= \sqrt{m^2 + (\sqrt{3}eA)^2}, \\
M_- &= \Delta
\end{aligned} \tag{2.4}$$

Table 2. The Higgs spectrum (a, b are color indexes, α, β are flavor indexes.)

Higgs multiplets	Representation under SM group	Mass matrix	From $SU(5)$ representation
T^a, \bar{T}_a	$(3, 1, -\frac{1}{3}), (\bar{3}, 1, \frac{1}{3})$	$\begin{pmatrix} 0 & -eA & -\frac{bb'A^2}{M_1} \\ 0 & m & -\frac{fb'A^2}{M_1} \\ -\frac{aa'A^2}{M_2} & 0 & \Delta \end{pmatrix}$	$5 + 45 + 5', \bar{5} + \bar{45} + \bar{5}'$
H^α, \bar{H}_α	$(1, 2, \frac{1}{2}), (1, 2, -\frac{1}{2})$	$\begin{pmatrix} 0 & \sqrt{3}eA & 0 \\ 0 & m & 0 \\ 0 & 0 & \Delta \end{pmatrix}$	$5 + 45 + 5', \bar{5} + \bar{45} + \bar{5}'$
$H_b^{a\alpha}, H_{a\alpha}^b$	$(8, 2, \frac{1}{2}), (8, 2, -\frac{1}{2})$	m	$45, \bar{45}$
$H^{a\alpha}, H_{a\alpha}$	$(3, 2, \frac{7}{6}), (\bar{3}, 2, -\frac{7}{6})$	m	$45, \bar{45}$
$H_{ab(s)}, H_{(s)}^{ab}$	$(\bar{6}, 1, -\frac{1}{3}), (6, 1, \frac{1}{3})$	m	$45, \bar{45}$
$H_\beta^{a\alpha}, H_{a\alpha}^\beta$	$(3, 3, -\frac{1}{3}), (\bar{3}, 3, \frac{1}{3})$	m	$45, \bar{45}$
H^a, \bar{H}_a	$(3, 1, -\frac{4}{3}), (\bar{3}, 1, \frac{4}{3})$	m	$45, \bar{45}$
Σ_b^a	$(8, 1, 0)$	$-\frac{2\sqrt{2}}{3}\lambda A$	75
$\Sigma_{b\beta}^{a\alpha}$	$(8, 3, 0)$	$-\frac{10\sqrt{2}}{3}\lambda A$	75
Σ_0	$(1, 1, 0)$	$\frac{4\sqrt{2}}{3}\lambda A$	75
$\Sigma_{\alpha(s)}^{ab}, \Sigma_{ab(s)}^\alpha$	$(6, 2, \frac{5}{6}), (\bar{6}, 2, -\frac{5}{6})$	$-\frac{4\sqrt{2}}{3}\lambda A$	75
$\Sigma^a, \bar{\Sigma}_a$	$(3, 1, \frac{5}{3}), (\bar{3}, 1, -\frac{5}{3})$	$\frac{8\sqrt{2}}{3}\lambda A$	75

and a pair of massless weak-doublets

$$H_U = H_5, \quad H_D = \bar{H}_{\bar{5}} \cos \theta - \bar{H}_{\bar{45}} \sin \theta \quad (2.5)$$

which are the two Higgs doublets of the MSSM. Here $\cos \theta = \frac{m}{M_+}$ and $\sin \theta = \frac{\sqrt{3}eA}{M_+}$.

3 Unification and Threshold Effects

In the GUT models the spectra are constrained by the threshold effects. In the present model, by requiring the three gauge couplings to unify at a scale Λ_{GUT} at 1-loop, we have

$$\begin{aligned}
(3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1})(m_z) &= \frac{1}{2\pi} \left\{ -2 \ln \frac{m_{SUSY}}{m_z} \right. \\
&\quad \left. + \frac{6}{5} \ln \frac{|\det M_T|^2 M_{H(s)}^9 M_{H_b}^4 M_{H_{a\alpha}}^4 M_{H_a}^7 M_{\Sigma_b}^5 M_{\Sigma_{\alpha(s)}}^{10} M_{\Sigma_a}^{10}}{m_z^2 M_+^2 M_-^2 M_{H_\beta}^{24} M_{\Sigma_{b\beta}}^{25}} \right\} \\
(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})(m_z) &= \frac{1}{2\pi} \left\{ 8 \ln \frac{m_{SUSY}}{m_z} + 6 \ln \frac{m_V^4 M_{H(s)}^9 M_{H_b}^4 M_{H_{a\alpha}}^6 M_{H_\beta}^6 M_{\Sigma_b}^a M_{\Sigma_{b\beta}}^{11}}{m_z^6 M_{H_{a\alpha}}^6 M_{H_a}^5 M_{\Sigma_{\alpha(s)}}^2 M_{\Sigma_a}^8} \right\},
\end{aligned} \quad (3.1)$$

where M_V is the mass of the X, Y gauge superfields. Numerically we need to include the running effects at 2-loop by adding approximately the corrections

$$\begin{aligned}
\delta_1^{(2)} &= -\frac{1}{4\pi} \sum_{j=1}^3 \frac{1}{b_j} (3b_{2j} - 2b_{3j} - b_{1j}) \ln \frac{\alpha_j(m_z)}{\alpha_5(\Lambda)}, \\
\delta_2^{(2)} &= -\frac{1}{4\pi} \sum_{j=1}^3 \frac{1}{b_j} (5b_{1j} - 3b_{2j} - 2b_{3j}) \ln \frac{\alpha_j(m_z)}{\alpha_5(\Lambda)},
\end{aligned} \quad (3.2)$$

on the R.H.S. of (3.1), respectively, where

$$b_i = \begin{pmatrix} \frac{33}{5} \\ 1 \\ -3 \end{pmatrix}, \quad b_{ij} = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix} \quad (3.3)$$

are the β -functions of gauge couplings in the MSSM at 1- and 2-loop level, respectively. The number $\alpha_5(\Lambda)$ can take simply its value at 1-loop level. The threshold corrections at the two-loop level are expected to be small and are thus omitted. Thus we have

$$\begin{aligned}(3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1})(m_z) &= \frac{1}{2\pi} \left\{ -2 \ln \frac{m_{SUSY}}{m_z} + \frac{12}{5} \ln \frac{x |\det M_T|}{m_z M_+ M_-} - \frac{1}{2} \sum_{j=1}^3 \frac{1}{b_j} (3b_{2j} - 2b_{3j} - b_{1j}) \ln \frac{\alpha_j(m_z)}{\alpha_5(\Lambda)} \right\}, \\ (5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})(m_z) &= \frac{1}{2\pi} \left\{ 8 \ln \frac{m_{SUSY}}{m_z} + 12 \ln \frac{ym_V^2 M_\Sigma}{m_z^3} - \frac{1}{2} \sum_{j=1}^3 \frac{1}{b_j} (5b_{1j} - 3b_{2j} - 2b_{3j}) \ln \frac{\alpha_j(m_z)}{\alpha_5(\Lambda)} \right\},\end{aligned}\tag{3.4}$$

where $M_\Sigma = \frac{10\sqrt{2}}{3}\lambda A$, while $x \sim 0.00006$ and $y \sim 2.73$ measure the mass splitting in 75 (see Table II).

The effect of the mass splitting at SUSY scale can be taken into account by replacing $\ln \frac{m_{SUSY}}{m_z}$ in eqs.(3.4) by [3]

$$\begin{aligned}-2 \ln \frac{m_{SUSY}}{m_z} &\rightarrow 4 \ln \frac{m_{\tilde{g}}}{m_{\tilde{w}}} + \frac{3}{5} \ln \frac{m_{\tilde{u}^c}^3 m_{\tilde{d}^c}^2 m_{\tilde{e}^c}}{m_Q^4 m_L^2} - \frac{8}{5} \ln \frac{m_{\tilde{h}}}{m_z} - \frac{2}{5} \ln \frac{m_H}{m_z}, \\ 8 \ln \frac{m_{SUSY}}{m_z} &\rightarrow 4 \ln \frac{m_{\tilde{g}}}{m_z} + 4 \ln \frac{m_{\tilde{w}}}{m_z} + 3 \ln \frac{m_Q^2}{m_{\tilde{u}^c} m_{\tilde{e}^c}}.\end{aligned}\tag{3.5}$$

The parameters can be also found in [3]. The difference between \overline{MS} -scheme and \overline{DR} -scheme can be found in Ref.[14]. Combining with the effect of top quark [15], we use the following formulas to replace $\frac{1}{\alpha_i}$ by

$$\frac{1}{\alpha_i} \rightarrow \frac{1}{\alpha_i} - \frac{C_i}{12\pi} + D_i \ln \frac{m_t}{m_z},\tag{3.6}$$

where $C_1 = 0, C_2 = 2, C_3 = 3$ and $D_1 = \frac{8 \cos^2 \theta_w}{15\pi}, D_2 = \frac{8 \sin^2 \theta_w}{9\pi}, D_3 = \frac{1}{3\pi}$.

Comparing the threshold effects with those in the MSGUT, we find an effective color-triplet Higgs mass

$$M_{H_c} = \frac{|\det M_T|}{M_+ M_-},\tag{3.7}$$

and an effective GUT scale

$$\Lambda_{GUT} = [m_V^2 M_\Sigma]^\frac{1}{3}.\tag{3.8}$$

Together with the extra x and y factors, they are constrained by [4]

$$\begin{aligned}5.8 \times 10^{18} GeV &\leq M_{H_c} \leq 6.0 \times 10^{19} GeV, \\ 1.2 \times 10^{16} GeV &\leq \Lambda_{GUT} \leq 1.4 \times 10^{16} GeV,\end{aligned}\tag{3.9}$$

for the wino mass $m_{\tilde{w}} = 200 GeV$, $\alpha_3(m_z) = 0.1185 \pm 0.002$, $\sin \theta_w(m_z) = 0.23117 \pm 0.00016$ and $\alpha^{-1}(m_z) = 127.943 \pm 0.027$. The uncertainties come mainly from the strong coupling constant. Note that a small x will enhance M_{H_c} to suppress proton decay.

The bound on M_{H_c} is about an order of magnitude larger than that in the model of [10], where the effective mass $m_T = \frac{m_{T_1} m_{T_2}}{m_\phi}$ and m_{T_1}, m_{T_2}, m_ϕ are about $10^{16} \sim 10^{17} GeV$ which constrains m_T to be $10^{15} \sim 10^{18} GeV$. In the present model with the $5' + \overline{5}'$ introduced, this constraint is release by (3.7) so that the effective color-triplet Higgs can be large.

The doublet-triplet splitting can be found in Table 3 for a set of representative parameters. Note that the existence of a pair of weak-doublets at $1.0 \times 10^{12} GeV$ implies that this is the $U_S(1)$ breaking scale, if we take $\frac{\Delta}{\langle S \rangle} \sim O(1)$ which is required by avoiding fine-tuning the couplings in (2.1).

Table 3. The Higgs, especially the doublet-triplet, spectrum at the GUT scale for $aA = a'A = bA = b'A = fA = 5 \times 10^{16} GeV$, $eA = 10^{16} GeV$, $M_1 = M_2 = 10^{18} GeV$ and $M_{H_c} = 5.8 \times 10^{18} GeV$.

Higgs multiplets	Masses
T^a, \bar{T}_a	$(0.25 \times 10^{16} GeV, 0.35 \times 10^{16} GeV, 1.41 \times 10^{16} GeV)$
H^α, \bar{H}_α	$(0, 1.0 \times 10^{12} GeV, 2 \times 10^{16} GeV)$
other Higgs from $45, \bar{45}$	$1 \times 10^{16} GeV$

4 Fermion Masses and Proton Decays

Without introducing extra particles, the matter fields are only the 10-plets ψ 's and $\bar{5}$ -plets ϕ 's. Their $U_S(1)$ and $U_P(1)$ quantum numbers are $(-\frac{h}{2}, -\frac{\tau}{2})$ and $(q + \frac{3}{2}h, \sigma + \frac{1}{2}\tau)$, respectively. The superpotential for the matter and the Higgs couplings is :

$$W_F = \sqrt{2}f_1^{ij}\psi_i^{\alpha\beta}\phi_{j\alpha}\bar{5}_\beta + \sqrt{2}f_2^{ij}\psi_i^{\alpha\beta}\phi_{j\gamma}\bar{45}_{\alpha\beta}^\gamma + \frac{1}{4}h^{ij}\epsilon_{\alpha\beta\gamma\delta\epsilon}\psi_i^{\alpha\beta}\psi_j^{\gamma\delta}5^\epsilon, \quad (4.1)$$

where i, j are generation indices. As we have discussed in the Introduction, the couplings of 45-plet Higgs and matter field are forbidden by the $U(1)$ symmetry. Denoting $\psi_i \ni Q'_i + u_i'^c + e_i'^c$ and $\phi_i \ni d_i'^c + L'_i$, we have

$$\begin{aligned} W_F \supset & Q'_i d_j'^c (f_1^{ij} \bar{H}_{\bar{5}} + \frac{1}{\sqrt{3}} f_2^{ij} \bar{H}_{\bar{45}}) + e_i'^c L'_j (f_1^{ij} \bar{H}_{\bar{5}} - \sqrt{3} f_2^{ij} \bar{H}_{\bar{45}}) + h^{ij} Q'_i u_j'^c H_5 \\ & + u_i'^c d_j'^c (f_1^{ij} \bar{T}_{\bar{5}} + f_2^{ij} \bar{T}_{\bar{45}}) + Q'_i L'_j (-f_1^{ij} \bar{T}_{\bar{5}} + f_2^{ij} \bar{T}_{\bar{45}}) - \frac{1}{2} h^{ij} Q'_i Q'_j T_5 + h^{ij} u_i'^c e_j'^c T_5 \end{aligned} \quad (4.2)$$

The couplings are related to the Yukawa couplings by

$$\begin{aligned} h &= Y_u, \\ f_1 &= \frac{1}{4 \cos \theta} (3Y_d + Y_e), \\ f_2 &= \frac{\sqrt{3}}{4 \sin \theta} (-Y_d + Y_e). \end{aligned} \quad (4.3)$$

Diagonalizing Y 's gives

$$\begin{aligned} Q'_i &= (u_i, V_{ij} d_j)^T, \quad u_i'^c = e^{-i\varphi_i} u_i^c, \quad e_i'^c = V_{ij} e_j^c \\ d_i'^c &= d_i^c, \quad L'_i = L_j = (\nu_j, e_j)^T, \end{aligned} \quad (4.4)$$

and

$$\begin{aligned} Y_u^{ij} &= h^i e^{i\varphi_i} \delta^{ij} = e^{i\varphi_i} \delta^{ij} \frac{m_{u_i}}{v_u}, \\ Y_d^{ij} &= V_{ij}^* \frac{m_{d_j}}{v_d}, \\ Y_e^{ij} &= V_{ij}^* \frac{m_{e_j}}{v_d}, \end{aligned} \quad (4.5)$$

where the mass parameters on the R.H.S. are mass eigenvalues, and V are the CKM matrix. Two of the three phases φ_i 's are independent[3].

In the new basics, we have

$$\begin{aligned} W_F \supset & m_u^i u_i u_i^c + m_d^i d_i d_i^c + m_e^i e_i e_i^c + e^{-i\varphi_i} u_i^c d_j^c (f_1^{ij} \bar{T}_1 + f_2^{ij} \bar{T}_2) + e^{-i\varphi_i} h^{ij} V_{jk} u_i^c e_k^c T_1 \\ & + Q_i L_j (-f_1^{ij} \bar{T}_1 + f_2^{ij} \bar{T}_2) - \frac{1}{2} h^{ij} Q_i Q_j T_1. \end{aligned} \quad (4.6)$$

The dimension-5 operators are

$$W_5 = C_{ijkl} (Q_i Q_j) (Q_k L_l) + D_{ijkl} (u_i^c e_j^c) (u_k^c d_l^c), \quad (4.7)$$

Table 4. Proton partial lifetimes. The experimental lower limits are typically around 10^{32-33} years[16].

Decay mode	lifetime of proton
$\tau(p \rightarrow K^+ + \bar{\nu}_\mu)$	$2.3 \times 10^{35} \sim 2.5 \times 10^{37} \text{ yrs}$
$\tau(p \rightarrow K^+ + \bar{\nu}_e)$	$4.7 \times 10^{36} \sim 5.0 \times 10^{38} \text{ yrs}$
$\tau(p \rightarrow K^+ + \bar{\nu}_\tau)$	$2.7 \times 10^{35} \sim 2.8 \times 10^{37} \text{ yrs}$
$\tau(p \rightarrow \pi^+ + \bar{\nu}_\mu)$	$4.7 \times 10^{35} \sim 5.0 \times 10^{37} \text{ yrs}$
$\tau(p \rightarrow \pi^+ + \bar{\nu}_e)$	$9.8 \times 10^{36} \sim 1.0 \times 10^{39} \text{ yrs}$
$\tau(p \rightarrow \pi^+ + \bar{\nu}_\tau)$	$6.9 \times 10^{35} \sim 7.3 \times 10^{37} \text{ yrs}$

where

$$C_{ijkl} = \frac{1}{2} h^{ij} [f_1^{kl} (M_T^{-1})_{11} - f_2^{kl} (M_T^{-1})_{12}] \quad (4.8)$$

for the LLLL operators, and

$$D_{ijkl} = h^{im} V_{mj} e^{-(\varphi_i + \varphi_k)} [f_1^{kl} (M_T^{-1})_{11} + f_2^{kl} (M_T^{-1})_{12}] \quad (4.9)$$

for the RRRR operators. Note that

$$\begin{aligned} (M_T^{-1})_{11} &= \frac{m\Delta}{\det M_T} \\ (M_T^{-1})_{12} &= \frac{\Delta}{\det M_T} eA, \end{aligned} \quad (4.10)$$

we have

$$\begin{aligned} C_{ijkl} &= \frac{1}{2M_{H_c}} Y_u^{ij} Y_d^{kl} \\ D_{ijkl} &= \frac{e^{-(\varphi_i + \varphi_k)}}{2M_{H_c}} Y_u^{im} V_{mj} (Y_d^{kl} + Y_e^{kl}). \end{aligned} \quad (4.11)$$

The operators will be dressed by the charginos to form the 4-fermion operators to calculate proton decay, as were usually done in the literature[3].

From equation (4.11), it is the effective Higgsino mass M_{H_c} which determines the coefficients of the dimensional-5 operators. A small x can enhance M_{H_c} through the threshold effects (3.4) and thus suppress the proton decay rates. This coincides with the observation from (2.2) that in a limit of vanishing Δ no proton decay can be driven by the dimensional-5 operators.

Numerically we take the Yukawa couplings h and $f_{1,2}$ to fulfill the data of fermion masses and mixing. The difference between D_{ijkl} in the minimal SU(5) model and that in our model is that m_{d_i} is now replaced by $\frac{m_{d_i} + m_{e_i}}{2}$. Following [17], the dominant mechanism for proton decay is through the wino dressed LLLL-type operators for $p \rightarrow K^+ + \bar{\nu}_{\mu(e)}$ and $p \rightarrow \pi^+ + \bar{\nu}_{\mu(e)}$, and through the higgsino dressed RRRR-type operators for $p \rightarrow K^+ + \bar{\nu}_\tau$ and $p \rightarrow \pi^+ + \bar{\nu}_\tau$. We list in Table 4 the proton partial lifetimes. Note that these partial lifetimes are generally of orders of magnitude longer than those in [10], as the effective color-triplet Higgs mass in (3.9) are much heavier than that in [10]. We found that these partial proton lifetimes are generally enhanced and the experimental data are satisfied.

5 Summary and Discussions

We have presented a renormalizable model of SUSY SU(5). The MPM is used to solve the doublet-triplet splitting problem while The GJM is used to describe the fermion masses and mixing. Two U(1) symmetries are used. The $U_P(1)$ is broken just below the Planck scale to give large masses to the 50's, so that below the GUT scale the evolutions of the gauge couplings are perturbative. The $U_S(1)$ is broken at a scale around 10^{12} GeV to enhance the effective mass of the color-triplet Higgs. At the meantime of describing the correct fermion masses and mixing, the proton decay rates are also suppressed.

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